# Math 464: Introduction to Dynamical Systems and Chaos
## Spring 2020

Tues/Thurs 1–2:15 pm    Hume 321

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| **Instructor:** | Samuel Lisi |
| **Office Hours:** | Hume 318, Tuesdays and Thursdays, 10 am – 11:30 am<br>Also available by appointment. |
| **Email:** | stlisi@olemiss.edu |
| **Textbook:** | (recommended, not required) *Nonlinear Dynamics and Chaos* by Strogatz.<br>Additional materials will be provided through Blackboard.<br>We will also use Mathematica quite extensively. |

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**Course content and objectives:**

This course is an introduction to dynamical systems, specifically with the goal of seeing the emergence of chaos in deterministic systems. This is a very broad topic with applications to many of the sciences. Our course will focus on the exploration of certain examples, numerically and analytically, but will downplay (without completely eliminating) proofs.

*Dynamics* is the study of the evolution of a system in time. For instance, given the knowledge of the current position and velocity of the satellites in orbit around the Earth, we might want to predict their future locations. Or, given some knowledge of the current population of various microbes in a sample, we would like to predict the future the population. Or, in a different context still, given the knowledge of the current composition of an archeological artifact, we would like to extrapolate backwards to understand its past.

Compared to the old stalwarts of calculus, geometry and algebra, the study of dynamical systems is relatively new. The discovery of (mathematical) *chaos* is itself even newer, a child of the mid-20th century. It has long been known that unpredictability can arise in a system where randomness appears. What has come as a surprise is that in actual fact, unpredictability arises in many contexts where there is no randomness. Laplace famously wrote that given a perfect knowledge of the present, and a sufficiently powerful computer/mind, the future can be entirely known:

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.
>
> Pierre Simon Laplace, *A Philosophical Essay on Probabilities* (1814)

The surprising discovery at the heart of the mathematical notion of chaos is that anything *less than perfect knowledge* is not good enough.

The course will cover a sampling of the following topics, very loosely following Strogatz's book (specifically, parts of Chapters 10, 11 and 2) with some supplemental material:

- 1-dimensional discrete dynamical systems, focusing on the example of the logistic map
- measurements of chaos (Lyapunov exponent)
- bifurcations, period doubling

- invariant sets, attractors and fractals
- universality (numerically) and Feigenbaum's constant
- numerical and analytical tools
- 1-dimensional continuous time dynamical systems (differential equations)
- bifurcations in continuous time dynamical systems
- modelling in applied mathematics — examples and some simple constructions.

**Objectives:** By the end of the course, the student will have

- learned the basic definitions and theorems in dynamical systems;
- learned the difference between discrete time and continuous time dynamical systems;
- learned how to apply these ideas to 1-dimensional systems;
- become aware of some of the subtleties and techniques of mathematical modeling;
- learned to construct their own model of a simple problem;
- improved their mathematical problem solving skills.

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**Assessment:**
The course grade will be based on the following items:

- Homework assignments. These will be assigned regularly and will count towards 20% of the course grade. The lowest homework grade will be dropped.
- Three midterm tests, held during class time. The lowest midterm test score will be replaced by the final exam, if that's better. These, averaged together, will count for 45% of the course grade. Tentatively, these will be held February 20, March 26 and April 21.
- Comprehensive Final Exam — 25% of grade. (Thursday, May 7th at noon.)
- End of term project — 10% of course grade.

Calculators, cell phones and other electronic equipment will **NOT** be permitted during tests. Students must show all work for each test question and arrive at a correct answer.

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**Term project:**
In a group of 2-3, you will prepare an end of term project about a topic of your choice related to the course material. For the project, your group will prepare two things: a written report (3-5 pages) and a 10-15 minute class presentation. The written report should be more detailed and technical than the presentation. Each member of the group should speak during the presentation.

We will discuss the expectations and goals of the project in more detail in class, after the first test. I will provide a list of suggested topics though you will be free to introduce your own. A good starting place is to consider some of the more difficult, open-ended problems in Strogatz's book.

The presentations will be in class on the 23rd and 28th of April.

In addition to questions about the regular course material, the final exam will have two open-ended questions, one asking for a reflection on your presentation topic and the other asking for a reflection about something interesting you might have learned or thought about in relation to a topic presented by someone else.

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**Mathematica**
Mathematica will play an important role in visualizing dynamical systems, and in some cases, making computations that are essentially impossible to do by hand. Mathematica is not the easiest piece of software to use – it is perhaps better to think of it as an interactive programming language

environment. Our class will meet in Weir Hall 104 on January 28th and 30th to go over the basics.

The university has a site license to Mathematica. The license allows all students to install and use Mathematica on their personal computers. Instructions for using the site license are listed on MyOleMiss.

If you are familiar with a competing piece of software (e.g. Matlab, Sage, python programming with whatever math libraries are available, ...), you are welcome to use this instead.

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**Blackboard:**
Blackboard will be used for course materials, homework and announcements.

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**Course Grade:**

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| 93 - 100 % | A | 83 - 86.9 % | B | 70 - 76.9 % | C |
| 90 - 92.9 % | A- | 80 - 82.9 % | B- | 60 - 69.9 % | D |
| 87 - 89.9 % | B+ | 77 - 79.9 % | C+ | 0 - 59.9 % | F |

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**Disability Access and Inclusion:** The University of Mississippi is committed to the creation of inclusive learning environments for all students. If there are aspects of the instruction or design of this course that result in barriers to your full inclusion and participation, or to accurate assessment of your achievement, please contact the course instructor as soon as possible. Barriers may include, but are not necessarily limited to, timed exams and in-class assignments, difficulty with the acquisition of lecture content, inaccessible web content, and the use of non-captioned or non-transcribed video and audio files. If you are approved through SDS, you must log in to your Rebel Access portal at `https://sds.olemiss.edu` to request approved accommodations. If you are NOT approved through SDS, you must contact Student Disability Services at 662-915-7128 so the office can: 1. determine your eligibility for accommodations, 2. disseminate to your instructors a Faculty Notification Letter, 3. facilitate the removal of barriers, and 4. ensure you have equal access to the same opportunities for success that are available to all students.
Note that unlike a few years ago, a student with testing accommodations must schedule all tests through the SDS Rebel Access online portal.

**Attendance, cheating and deadlines:**
Attendance is mandatory but will not be checked.

Cheating on any exam, quiz, homework, work to be completed in class; theft, or attempted theft of exam questions; or possession of exam questions prior to the time for examination shall all be offenses subject to appropriate penalties.

The course withdrawal deadline is Monday, March 2nd. After the Course Withdrawal Deadline, courses dropped will be recorded on University records and the W grade will be recorded if the student is not failing the course at the time of withdrawal; otherwise, the grade recorded will be F. After the course withdrawal deadline, a student may drop a course only in cases of extreme and unavoidable emergencies as determined by the academic dean. (Change of major or dissatisfaction with an expected grade are not acceptable reasons.)