Local Limit Theorem for Linear Random Fields

Abstract

In this talk, we investigate the conditions under which a local limit result holds for the linear random field

\[ X_j = \sum_{i \in \mathbb{Z}^d} a_i \varepsilon_{j-i}, \]

defined on \( \mathbb{Z}^d \) with absolutely continuous innovations \( \varepsilon_i \) that are independent and identically distributed with mean zero and finite variance. Let \( \Gamma_n^d \) be a sequence of subsets of \( \mathbb{Z}^d \). Define the sum \( S_n = \sum_{j \in \Gamma_n^d} X_j \), and let \( B_n^2 = \text{Var}(S_n) \). Building on Shore’s previous work, we are able to show that the local limit theorem holds for both short and long memory linear random fields in the sense that the sequence of measures \( \sqrt{2\pi B_n} P(S_n \in (a,b)) \) converges to Lebesgue measure. That is,

\[ \lim_{n \to \infty} \sqrt{2\pi B_n} P(S_n \in (a,b)) = b - a, \]

assuming minimal and reasonable requirements of the innovations and of the sets \( \Gamma_n^d \).

This is a joint work with Magda Peligrad and Hailin Sang.