Number Theory Seminar

Friday, October 4th, 2019

11:30 am in Hume 321

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Subspaces in difference sets and Möbius randomness

ABSTRACT

This talk will consist of two seemingly unrelated topics in mathematics. A common theme in analysis, combinatorics and number theory states that if A is a large subset of a certain group G, then the difference set $A - A$ contains nice structures. For example, Steinhaus' theorem says that if A is a subset of positive Lebesgue measure of $\mathbf{R}$, then $A - A$ contains an interval around 0. We are interested in this phenomenon in vector spaces over a finite field. If A is a subset of positive density of $\mathbf{F}_p^n$, then $A - A$ must contain a large subspace. Furthermore, the more sums or differences we take (e.g. $A + A - A - A$), the larger subspaces we are guaranteed to find. This is the content of Bogolyubov's theorem. I will talk about a bilinear analog of this theorem.

The Möbius function is one of the most important functions in number theory. However, its behavior is random-like in many respects. I will talk about a function field instance of this principle, namely that the Möbius function over $\mathbf{F}_q[t]$ does not correlate with linear or quadratic phases. Curiously, our main tool in establishing this is the bilinear Bogolyubov theorem above. This is joint work with Pierre-Yves Bienvenu.

This talk will be accessible and students interested in number theory, combinatorics and analysis are encouraged to attend.