New Results on Polynomials in Difference Sets

ABSTRACT

It is a well known result, established independently by Sárközy and Furstenberg, that a set of integers with positive upper density must contain two distinct elements that differ by a perfect square. The best-known quantitative upper bounds for this result were established with an intricate Fourier analytic argument by Pintz, Steiger, and Szemerédi. In the first half of this talk, we discuss the extension of these bounds from perfect squares to, at long last, the largest possible class of single-variable polynomials. In the second half, we discuss even better upper bounds for a large class of two-variable polynomials. In both settings, we utilize a sieve as a bridge to algebraic geometry, gaining access to optimal exponential sum estimates over finite fields.