Semidualizing modules give a defective Gorenstein defect

Semidualizing modules arise in the study of dualities over commutative rings. Here a finitely generated R-module C is *semidualizing* provided that $\mathrm{Hom}_R(C, C) \cong R$ and $\mathrm{Ext}^i_R(C, C) = 0$ for all $i \geq 1$. Examples include the free R-module of rank 1 and Grothendieck's dualizing module for R (if one exists). Applications of these examples include Auslander and Bridger's G-dimension and Grothendieck's local duality.

It has been argued that the number $s_0(R)$ of (isomorphism classes of) semidualizing modules over a local ring R is a measure of the severity of the singularity of R, namely, how far R is from being Gorenstein. For instance, if R is Gorenstein, then $s_0(R) = 1$; and the converse holds if R is Cohen-Macaulay with a dualizing module. In this talk, however, we will show that s_0 does not satisfy a standard condition one usually expects from such a measure: this invariant can increase after localizing. This is joint work with Saeed Nasseh, Ryo Takahashi, and Keller VandeBogert. Much of the talk will be spent on examples and big-picture perspective accessible to graduate students.