The topic of this expository talk can best be explained with an example. Let $C[0, 1]$ denote the space of all continuous functions $f: [0, 1] \to \mathbb{R}$. Consider the subset of $C[0, 1]$ consisting of those functions which are nowhere differentiable, together with the zero function. It has been known for nearly two centuries that such functions exist, and in fact are very “plentiful.” Our point is that, in addition, the set of such functions is lineable, i.e. it contains an infinite dimensional vector space.

As we will indicate, this type of situation—with an “unpleasant” set, contained in a “decent” one, turning out to contain large subsets with excellent linear structure—is ubiquitous. Our plan is to describe several such situations and to also indicate the few known cases where this does not occur.