Colloquium Talk

Friday, November 7, 2014 3:00-3:50 pm in Hume 331

Exact solution of the six-vertex model

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We will discuss certain aspects of the six-vertex model with domain wall boundary conditions (DWBC) formulated on a square lattice with N^2 vertices. This model is a prototypical "ice-model" in statistical mechanics that exhibits connections to purely combinatorial problems such as the Alternating sign matrix conjecture and domino tilings of the Aztec diamond. For finite N the model was solved by Izergin and Korepin through the application of the Yang-Baxter equations – their work lead to an exact formula for the partition function Z_N in terms of an $N \times N$ Hankel determinant. Subsequently Paul Zinn-Justin re-expressed the Hankel determinant in terms of the partition function of a random matrix model with non polynomial interaction. Based on his observation we can thus connect the partition function Z_N to orthogonal polynomials and analyze the model in the thermodynamical limit as $N \to \infty$. On the technical level we employ the Riemann-Hilbert approach and obtain leading and subleading terms in the asymptotics of the partition function in all phase regions, and also on the separating lines between the phases.

This is joint work with Pavel Bleher.