Combinatorics and Graph Theory Seminar

Wednesday Oct 29th, 2014

3:00 pm-3:50 pm in Hume 322

On a conjecture of Leader and Radcliffe related to the Littlewood-Offord problem

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ABSTRACT

The classical Littlewood-Offord problem is a combinatorial question in geometry that asks for the maximum number subsums of vectors $v_1, \dots, v_n \in \mathbb{R}^d$ of length at least 1 that fall into a fixed set $A \in \mathbb{R}^d$. Erdős proved that the best upper bound in the case $d = 1$ and $A = (x, x+2]$ is

$$\binom{n}{\lfloor \frac{n}{2} \rfloor}$$

using Sperner's theorem. The problem has a very natural probabilistic formulation. Consider n independent random variables ε_i such that $\mathbb{P}(\varepsilon_i = \pm 1) = 1/2$ and let $|a_i| \geq 1$. Then

$$\sup_{a_i, x} \mathbb{P}(a_1\varepsilon_1 + \cdots + a_n\varepsilon_n \in (x, x+2]) = \mathbb{P}(\varepsilon_1 + \cdots + \varepsilon_n \in (-1, 1]).$$

In this talk I shall discuss a strong generalization of the latter inequality for arbitrary random variables with only information about their local concentration provided. In particular, a proof of a question posed by Leader and Radcliffe will be presented. One of the main ingredients of the proof will be a Sperner type theorem for multisets. Even in the case for sets the proof is new and very elementary.

Keywords: Littlewood-Offord problem, Sperner's theorem, concentration function