

Colloquium

Friday, October 10, 2014, 3:00-3:50 pm in Hume 331.

Random Matrix Models, Non-intersecting random paths, and the Riemann-Hilbert Analysis

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Random matrix theory (RMT) is a very active area of research and a great source of exciting and challenging problems for specialists in many branches of analysis, spectral theory, probability and mathematical physics. The analysis of the eigenvalue distribution of many random matrix ensembles leads naturally to the concepts of determinantal point processes and to their particular case, biorthogonal ensembles, when the main object to study, the correlation kernel, can be written explicitly in terms of two sequences of mutually orthogonal functions.

Another source of determinantal point processes is a class of stochastic models of particles following non-intersecting paths. In fact, the connection of these models with the RMT is very tight: the eigenvalues of the so-called Gaussian Unitary Ensemble (GUE) and the distribution of random particles performing a Brownian motion, departing and ending at the origin under condition that their paths never collide are, roughly speaking, statistically identical.

A great challenge is the description of the detailed asymptotics of these processes when the size of the matrices (or the number of particles) grows infinitely large. This is needed, for instance, for verification of different forms of "universality" in the behavior of these models. One of the rapidly developing tools, based on the matrix Riemann-Hilbert characterization of the correlation kernel, is the associated non-commutative steepest descent analysis of Deift and Zhou.

Without going into technical details, some ideas behind this technique will be illustrated in the case of a model of squared Bessel nonintersecting paths.