

# Combinatorics Seminar

Wednesday, April 14, 2004

3:00 pm in Hume 331

(Refreshment will be served at 2:40pm in Hume 307)

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## Plane Graphs with Positive Curvature

### ABSTRACT

Let  $G$  be a plane graph (finite or infinite) such that (1)  $G$  is locally finite and (2) every face of  $G$  is bounded by a cycle. Then the combinatorial curvature of  $G$  is the function  $\Phi(G) : V(G) \rightarrow R$  such that for any  $x \in V(G)$ ,  $\Phi(x) = 1 - d(x)/2 + \sum_{x \in F} 1/|F|$ , where the summation is taken over all facial cycles of  $G$  containing  $x$ . The curvature interprets the degree of difficulty of tiling the plane at  $x$  and it is dual of another curvature introduced by Gromov. Higuchi proved that there is a negative real number  $\mu$  such that  $\Phi(x) < \mu$  if  $\Phi(x) < 0$  and the positive curvature can be arbitrarily small. We show that if  $\Phi(x) \geq 0$  then  $\sum_{x \in V(G)} \Phi(x)$  is bounded if and only if there are only finite number  $x$  such that  $\Phi(x) \neq 0$ . Higuchi also conjectured that  $G$  is finite if  $\Phi(x) > 0$  for all  $x$ . Sun and Yu proved this for cubic plane graphs. We completely characterized finite graphs with positive curvatures provide the number of vertices is large.