Maximal Class Numbers of CM Number Fields

ABSTRACT

Through the class number formula, the Generalized Riemann Hypothesis (GRH) can be used to formulate upper bounds for the class numbers of certain natural families of number fields. A natural question to ask is whether or not these bounds are sharp. While the answer is known to be positive for all natural families of quadratic and cubic number fields, it is actually negative for certain families of quartic number fields. More generally, it turns out that for any family of CM number fields the GRH conditional upper bound on the class number is never attainable. However, by subdividing these natural families of CM number fields according to their maximal real subfield an improved upper bound can be formulated, and we will see that, at least under the assumption of the GRH, this new bound is sharp.