

Title: Rings of real analytic and real entire functions

Abstract:

Let  $E(\mathbb{C})$ ,  $E(\mathbb{R})$ , and  $A(\mathbb{R})$  denote respectively, the ring of entire functions, the ring of real entire functions, and the ring of real analytic functions. In 1952, I showed that for any maximal ideal  $M$  of  $E(\mathbb{C})$ , then  $E(\mathbb{C})/M$  is isomorphic to the complex field  $\mathbb{C}$  even though it sometimes is infinite dimensional as an algebra over  $\mathbb{C}$ . If  $M$  is a maximal ideal of  $A(\mathbb{R})$ , then  $A(\mathbb{R})/M$  is either  $\mathbb{C}$ ,  $\mathbb{R}$ , or is a particular kind of non-Archimedean real-closed field containing  $\mathbb{R}$ . If  $M$  is a maximal ideal of  $E(\mathbb{R})$ , then  $E(\mathbb{R})/M$  can be one of these latter three fields, but it is an open problem whether these are the only such fields.