Dicromatic number and fractional chromatic number

ABSTRACT

Given an undirected graph $G$, the chromatic number $\chi(G)$ is the minimum order of partitions of $V(G)$ into independent sets. Given a directed graph $D$, a vertex set is acyclic if it does not contain a directed cycle. The chromatic number $\chi(D)$ is the minimum order of partitions of $V(G)$ into acyclic sets. The dichromatic number of an undirected graph $G$, denoted by $\overrightarrow{\chi}(G)$, is the maximum chromatic number over all its orientations. Erdős and Neumann-Lara proved that $C_1 \frac{n}{\log n} \leq \overrightarrow{\chi}(K_n) \leq C_2 \frac{n}{\log n}$ for some constants $C_1, C_2$. They conjectured that if the dichromatic number of a graph is bounded, so does its chromatic number.

Let $\mathcal{I}(G)$ be the set of all independent sets of $G$, and let $\mathcal{I}(G, x)$ be the set of all those independent sets which include vertex $x$. For each independent set $I$, define a nonnegative real variable $x_I$. The fractional chromatic number $\chi_f(G)$ is the minimum value of $\sum_{I \in \mathcal{I}(G)} x_I$, subject to $\sum_{I \in \mathcal{I}(G, x)} x_I \geq 1$ for each vertex $x$. We prove that there is a constant $C$ such that for any graph $G$, we have $\overrightarrow{\chi}(K_n) \geq C \frac{N}{\log n}$.

This is joint work with Professor Bojan Mohar in Simon Fraser University.