Bregman distance, approximate compactness and Chebyshev sets in Banach spaces

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Abstract:

In this paper, we first introduce the notion of locally uniformly totally convex functions defined on a Banach space and discuss its relations to totally convex, essentially strictly convex, and uniformly convex functions. We then present some sufficient conditions for the (norm-weak) upper semicontinuity and the (norm-weak) continuity of the Bregman projection operator \( g \subset C \) in terms of the notion of \( D \)-approximate (weak) compactness whenever \( g \) is either locally uniformly totally convex function or coercive, and \( C \) is a nonempty closed subset of \( \text{int}(\text{dom}g) \). We finally present certain sufficient conditions as well as equivalent conditions for the convexity of a Chebyshev (in the sense of Bregman distance) subset of a Banach space \( X \). Our results extends the corresponding results of [Bauschke, et al., J. Approx. Theory, doi:10.1016/j.jat.2008.08.014] to infinite dimensional spaces.