ANALYSIS SEMINAR

Vector Measures Without Weak Compactness

Iwo Labuda

Department of Mathematics, University of Mississippi

THURSDAY SEPTEMBER 16, 2010 AT 8:00 AM IN HUME 321

Abstract: *The title above refers to a 1955 paper of Bartle, Dunford and Schwartz 'Weak compactness and vector measures' in which integration of scalar functions with respect to a vector measure was first defined. Vector measures considered in the original paper were Banach space valued and the notion of weak compactness played the key role in the development of the theory. 55 years later, I am happy to report that the theory can be developed without any constrains on the range space (and so the notion of weak topology is not even available, in general). Let $\mathcal{A}$ be a σ–algebra of subsets of a set T, X be a Hausdorff sequentially complete topological vector space, and let $\mu : \mathcal{A} \to \mathcal{X}$ be a countably additive measure. Denote by $L^0(\mu)$ the space of (classes of) μ–measurable $\overline{\mathbb{R}}$–valued functions. Assume that the measure μ is convexly bounded or, equivalently, all μ–essentially bounded functions are μ–integrable. Then the classical definition of the Bartle-Dunford-Schwartz integral is possible. The space $L^1(\mu)$ of BDS- integrable functions is a vector lattice, but with its natural topology τ^- is only an $\mathcal{A}$–solid topological vector group and, in general, the Lebesgue Dominated Convergence Theorem is not valid in $L^1(\mu)$. Let $L^1_o(\mu)$ be the largest vector subspace of $L^1(\mu)$ that is solid in $L^0(\mu)$. $L^1_o(\mu)$ with its natural topology τ^o is a Dedekind σ–complete Hausdorff locally solid vector lattice (F–lattice, if X is an F–space) having the σ–Lebesgue property (the Dominated Convergence Theorem holds). If X contains no isomorphic copy of c_0, then $L^1_o(\mu)$ has the σ–Levi property (the Beppo Levi Theorem holds).*