Analyzing Seminar
Compact Families of Sets

Iwo Labuda
Department of Mathematics, University of Mississippi

Thursday, September 21, 2005, in Room 302 at 12:00 noon

Abstract:
A topological space $X$ is said to be compact if every filter base on $X$ has a cluster point (Vietoris, 1920) or, equivalently, every open cover of $X$ has a finite subcover (Alexandrov and Urysohn, 1922).

Many other notions of compactness arise in topology and analysis. To name a few, people investigate spaces that are countably compact, sequentially compact, Lindelöf (this also is a compactness type property), paracompact, metacompact, Eberlein compact, angelic, pseudocompact, feebly compact and on... and on...

If $K \subset X$, then $K$ is a compact (in any sense) subset of $X$ whenever $K$ as a subspace of $X$ is compact. Let now $K$ be family of subsets of $X$. What it would mean that $K$ is compact? Can we have a common principle that covers (at least a fair number of) the definitions given above and applies to the families of subsets? It looks that I now know the answers.

I will discuss the notions of $\mathbb{P}/\mathbb{R}$-compact (midcompact, ultracompact) at a family $\mathcal{B}$ of sets and the role of filter $D$-compactness as a unifying scheme.