Interview Talk

NONCOMMUTATIVE SYMMETRIC SPACES

OF MEASURABLE OPERATORS

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Abstract: *Let $\mathcal{M}$ be a semifnite von Neumann algebra with a faithful, normal, semifnite trace τ , and E be a symmetric Banach function space on $[0;\tau(1))$. The symmetric space $E(\mathcal{M};\tau)$ of τ–measurable operators consists of all τ–measurable operators x for which the singular value function $\mu(x)$ belongs to E and is equipped with the norm $\|x\|_{E(\mathcal{M},\tau)} = \|x\|_E$. Special cases of noncommutative symmetric spaces are the following well known spaces: $L_p(\mathcal{M};\tau)$–noncommutative L_p–spaces; unitary matrix spaces C_E; Schatten spaces C_p, in particular trace class C_1 and the class of Hilbert–Schmidt operators C_2. The noncommutative spaces have been investigated by several mathe- maticians as J. Arazy, P. Dodds, B. De Pagter, G. Pisier, Y. Raynaud, F. Sukochev, and Q. Xu. We will discuss some geometric properties of $E(\mathcal{M};\tau)$, like complex uniform rotundity, smoothness, and Fréchet smoothness. Below there are examples of our results.*

Theorem 0.1. Let E be a symmetric space on $[0;\alpha), \alpha = \tau(\mathbf{1})$ and $\mathcal{M}$ be a semifnite von Neumann algebra with a faithful, normal, σ–fnite trace τ . An operator x is a complex extreme point of $B_{E(\mathcal{M}.\tau)}$ if and only if $\mu(x)$ is a complex extreme point of B_E and one of the following, not mutually exclusive, conditions holds:
(i) $\mu_\infty(x) = 0$
(ii) $n(x)\mathcal{M}n(x^\star) = 0$ *and* $|x| \geq \mu_\infty(x))s(x)$.

Theorem 0.2. Let $\mathcal{M}$ be a semifnite von Neumann algebra with a normal, faithful, semifnite trace τ . The symmetric function space E on $[0;\tau(\mathbf{1}))$ is complex uniformly rotund if and only if $E(\mathcal{M};\tau)$ is complex uniformly rotund.