ANALYSIS SEMINAR

Domination by Second Countable Spaces

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Abstract: *Let K be a compact space and let $\Delta : \{(x,x) : x \in K\} \subset K \times K$ be its diagonal. Taking as a reference the known fact that K is metrizable if, and only if, the complement of the diagonal $(K \times K) \setminus \Delta$ is an $\mathcal{F}_\sigma$ in $K \times K$, we move to the more intriguing case when $(K \times K) \setminus \Delta = \bigcup \{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ where each A_α is compact and $A_\alpha \subset A_\beta$ whenever $\alpha \leq \beta$. We prove that the latter assumption also implies metrizability when either $\{A_\alpha : \alpha \in \mathbb{N}^{\mathbb{N}}\}$ is a fundamental family of compact subsets for $(K \times K) \setminus \Delta$ or when $MA(\omega_1)$ is assumed. The success when proving these results relies upon the generation of usco maps: if we want to say it this way, it relies in some sort of understanding of compactoid filters. We provide applications (old and new) of the results and techniques presented here to functional analysis: metrizability of compact subsets in inductive limits, Lindelöf property of WCG Banach spaces, separability of Frchet-Montel spaces, Lindelöf-Σ character of spaces $C_p(X)$, etc.*

Students are welcome.