Algebra & Number Theory Seminar

Wednesday, February 15th, 2017

3:00 pm in Hume 101

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A survey of the polynomial method in arithmetic combinatorics

ABSTRACT

A basic quest in arithmetic combinatorics is to bound the size of a set of integers between 1 and N containing no 3-term arithmetic progression. This has been intensively studied since 1946 with some success and a lot of frustration. The same question can be asked in $\mathbf{F}_3^n$ instead of the integers; there, a 3-term progression can also be seen as 3-points on a line. The same (analytic) method produced similar or better results, but last May, a radically new and incredibly simple method dramatically lowered the bound, passing from $3^n/n$ to a stunning $(2.78)^n$. This was due to Croot-Lev-Pach and Ellenberg-Gijswijt. Their method uses polynomials, more precisely simple facts about the dimension of spaces of polynomials. This is vaguely reminiscent of other problems whose finite fields variants were successfully attacked with some form of "polynomial method". In this talk I will discuss the principle of this method and survey the wealth of results that it has produced. This includes my own about generalizations of Roth's theorem to function fields.