Math 464: Introduction to Dynamical Systems and Chaos
Spring 2015

MWF 10:00-10:50   Hume 203

Instructor:	Samuel Lisi
Office Hours:	Hume 318, Monday 2:30–4:30; Wednesday 9–10.
	Also available by appointment.
Email:	stlisi@olemiss.edu

Textbook:	*Nonlinear Dynamics and Chaos* by Strogatz.
	I will also provide additional documentation on Blackboard.
	We will also use software for visualization and calculations.

Course content and objectives:

This course is an introduction to dynamical systems, specifically with the goal of seeing the emergence of chaos in deterministic systems. This is a very broad topic with applications to many of the sciences, so we will necessarily only see a selection of topics, to give a taste of the beautiful mathematics that is out there. Our course will focus on the exploration of certain examples, numerically and analytically, but will downplay (but not completely eliminate) proofs.

The term *dynamics* refers to the study of the evolution of a system in time. For instance, given the knowledge of the current position and velocity of the satellites in orbit around the Earth, we might want to predict their future location. Or, given some knowledge of the current population of various microbes in a sample, we would like to predict the future behavior of this population. Or, in a different context still, given the knowledge of the current composition of an archeological artifact, we would like to extrapolate backwards to understand its past.

Compared to the old stalwarts of calculus, geometry and algebra, the study of dynamical systems is relatively new. The discovery of and then study of (mathematical) *chaos* is itself even newer, a child of the mid-20th century. It has long been known that unpredictability can arise in a system where randomness appears. What has come as a surprise is that in actual fact, unpredictability arises in many contexts where there is no randomness. Laplace famously wrote that given a perfect knowledge of the present, and a sufficiently powerful computer/mind, the future can be entirely known:

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.
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> Pierre Simon Laplace, *A Philosophical Essay on Probabilities* (1814)

However, the dramatic discovery involved in the mathematical notion of chaos is that anything less than perfect knowledge is not good enough to know anything. This was foreshadowed in the 19th century by the difficulty of reconciling the time reversibility of Newton's Laws and the 2nd law of Thermodynamics.

The course will cover a sampling of the following topics, very loosely following Strogatz's book with some supplemental material. We will see how dynamical systems arise from maps and from systems of differential equations. We will see some of the theory of non-linear systems of differential equations, and use it to build up to some chaotic examples, notably Lorenz's famous attractor and the "butterfly effect". We will also consider some 1-dimensional maps, notably the logistic map, and how chaos emerges in some cases, both numerically and analytically. We will quantify exactly what we mean by chaos in this context (Lyapunov exponent) and numerically study some of the strange unifying behavior exhibited by all maps with similar properties (universality). Some of the examples we consider will come from mathematical models, and in some cases, we will discuss the construction of these models.

By the end of the course, the student will have

- learned the basic definitions and theorems in dynamical systems;
- learned how to apply these to continuous time systems in 1 and 2 dimensions and to discrete systems on the real line;
- become aware of some of the subtleties and techniques of mathematical modeling;
- learned to construct their own model of a simple situation;
- improved their mathematical problem solving skills.

Assessment: The course grade will be based on the following four items:

1. Three midterm tests, held during class time. The lowest midterm test score can be replaced by the final score, if that's better. These, averaged together, will count for 40% of the course grade. Tentatively, these will be held February 20, March 24 and April 24.
2. Comprehensive Final Exam — 25% of grade. (Friday, May 12th at 8 AM.)
3. Homework assignments (approximately 6). These will be assigned regularly and will count towards 20% of the course grade. The lowest homework grade will be dropped.
4. End of term project — 10% of course grade.
5. Participation — 5% of course grade. This will take attendance and class participation into account.

Calculators, cell phones and other electronic equipment will **NOT** be permitted during quizzes, tests or exams. Students must show all work for each test question and arrive at a correct answer.

Term project:
In a group of 2-3, you will prepare an end of term project about a topic that interests you. For the project, your group will prepare two things: a written report (3-5 pages) and a 10-15 minute class presentation. The written report should be more detailed and technical than the presentation. (Each member of the group should speak during the presentation.)

We will discuss the expectations and goals of the project in more detail in class. In particular, I will provide a list of suggested topics though you will be free to introduce your own.

The presentations will start on the 26th of April.

The final exam will have two open-ended questions, one asking for a reflection on your presentation topic and the other asking for a reflection about something interesting you might have learned or thought about in relation to a topic presented by someone else.

Visualizations software:
I will use Mathematica worksheets frequently in class to help visualize various topics and constructions. I will make these available either through Blackboard or through Box. A small amount of the homework will benefit from using Mathematica or other software. If you are unfamiliar with Mathematica or struggle with getting it to work correctly, please do see me in office hours (or make an appointment) for us to go over the basics.

Matlab is also a good choice for this. If you are interested in applied math or engineering, I recommend taking the time to learn how to use Matlab for our visualizations.

There are many other software tools used to visualize and make calculations for dynamical systems. You are welcome to use any of these, though I am much less familiar with them.

Note that the university has site licenses to **both** Matlab and Mathematica so they are **free** for you to install and use.

Blackboard:
Blackboard will be used for course materials, homework and announcements.

Academic needs: It is the responsibility of any student with a disability who requests a reasonable accommodation to contact the Office of Student Disability Services (915-7128). Contact will then be made by that office through the student to the instructor of this class. The instructor will then be happy to work with the student so that a reasonable accommodation of any disability can be made.

Course Grading Scale

90 - 100 %	A
87 - 89 %	A-
85 - 86 %	B+
80 - 84 %	B
77 - 79 %	B-
75 - 77 %	C+
70 - 75 %	C
60 - 69 %	D
0 - 59 %	F