Combinatorics Seminar

Monday, Sept. 17, 2012
2:00 pm in Hume 331

Nearly Tight Linear Programming Bounds for Demand Matching in Bipartite Graphs

Dr. Hehui Wu

School of Computer Science
McGill University, Canada

ABSTRACT

We consider the demand matching problem which is a generalization of the maximum weight bipartite matching problem as well as the b-matching problem. We are given a simple bipartite graph $G = (V, E)$, a demand function d and a profit function π on edges and a capacity function b on vertices. A subset M of edges is called a demand matching if the sum of demands d_e of edges chosen in M incident at v is at most b_v for each vertex v. The goal of the demand matching problem is to select a demand matching M which maximizes the sum of profit of edges in M. When all demands $d_e = 1$, this problem is exactly the b-matching problem.

We give nearly tight upper and lower bounds on the integrality gap of a natural linear programming relaxation for the problem. Our first result is to show that the integrality gap is bounded from above by the fractional coloring number of a *tree-net*. A tree-net is a graph obtained by connecting non-adjacent vertices of a tree by vertex disjoint paths of length at least two. We then present an explicit bound of 2.709 on the fractional chromatic number of any tree-net which also results in a 2.709-approximation algorithm. To complement this algorithm, we explicitly show a lower bound of 2.699 on the integrality gap by constructing tree-net graphs whose fractional chromatic number is at least 2.699.

This is joint work with Dr. Mohit Singh.